Handling shelf space with optimal EOQ model

Abstract

Ideally, we want to have single-trip in-store replenishment. Namely, only a single trip is needed for a store worker to travel to fill the shelf after a product is received at the store. This can be achieved by having the lot size received smaller than the shelf space less the number of units left on the shelf at the shelf replenishment time. In this paper, we introduce the consideration of the in-store handling cost by the second trip in an EOQ-type model to determine the optimal order quantity, given the retail shelf space. Our model considers the case where products are ordered in multiple of packs and there is a part of distribution-handling cost that is per each pack handled. The insights gained from the model will help managers both in retailers and vendors or manufacturers make better decisions with regards to the lot size produced, ordered, and delivered in the supply chain.

Keywords: Retail replenishment, EOQ, Inventory control, Shelf space allocation, Store handling, Distribution handling

1. Introduction

With the increasing competition in the retail industry, retailers are looking for ways to improve its operations, especially, the replenishment processes. Managing the replenishment process, which is the core process in managing the retail supply chain, efficiency is crucial in controlling costs while improving services to customers. The retail replenishment process starts from ordering products to putting the products on shelves. In an empirical study by Saghir and Jonson (2001), 75% of the handling time in the replenishment process is found to occur in store.

Therefore, it is important that we account for in-store handling and its related costs as part of decision models for improving the replenishment process. However, little attention has been given to account for the cost of the in-store handling (Zelst *et al*., 2009).

The in-store handling includes the process from receiving products from a delivery truck and, depending on the store's policy and timing of the delivery and shelf-fill operation, the products are then moved to either sales floor or the store's backroom.

(1) In the first case, the products are moved to the sales floor for filling shelves. Then, after the shelves are filled, units of products will be moved to the store's backroom for storage if not all the units can fill the shelves.

(2) In the second case, the products are moved to the backroom for storage and will later be moved to the sales floor for filling shelves. Then, like the first case, after the shelves are filled, units of products will be moved to the store's backroom for storage if not all the units can fill the shelves.

In both cases, any stored units in case not all the units can fill the shelves will later be moved back to the sales floor for filling the shelves; namely a second trip is required.

Since retailers, especially those operating large discount stores, carry thousands of stock-keeping units (SKUs) and have thousands of square meters of sales floor space, the in-store handling activities are significant. These activities can be reduced substantially, if units of products do not need to be moved back to the store's warehouse for storage and later moved back to the sales floor because not all the units can fill the shelves. The decisions on the ordering and shelf space, the number of product units that can fill the shelf, are the major contributing factors for whether the extra trip to move the product units from the warehouse back to the sales floor and fill the shelf will be needed.

There are studies in the literature that attempt to include the assortment and shelf space allocation problems in the inventory decision-making process. Corstjens and Doyle (1981) develop a the shelf space allocation model using nonlinear programming model to maximize the profit subject to available supply and minimum and maximum space allowed for each item. Urban (1998) develops models that integrate assortment, allocation, and replenishment decisions. The models explicitly consider that the demand rate is a function of inventory level. Several other papers also discuss the inventory-dependent demand, including Mandel and Phaunjdar (1989), Datta and Pal (1990), Urban (1992), Pal *et al*. (1993), Wang (1994), Urban (1995), Giri *et al*. (1996), Khmelnitsky and Gerchak (2001), Gerchak and Wang (1994), and Wang and Gerchak (2001). Hariga *et al*. (2007) proposes an optimization model to determine the product assortment, inventory, display area, and shelf space allocation decisions that maximize the profit subject to shelf space and backroom storage constraints. The model explicitly distinguishes between on-shelf or sales floor and backroom inventories. However, none of these models take into account the in-store handling activities, specifically the second-trip required when not all units can fill the shelf.

In attempts to model buyer-vendor coordination for replenishment decisions, several papers extend the Economic Order Quantity (EOQ) model to consider the quantity-related cost function such as quantity-price discount and quantity-freight cost discount. These papers, including Goyal (1976), Monahan (1984), Lee (1986), Lee and Rosenblatt (1986), Tersine and Barman (1994), Corbett (2000), Toptal *et al*. (2003), and Toptal (2009), consider the cases where there is a change in replenishment costs when the order quantity changes. However, they do not consider the in-store handling cost resulted from the decision on the order quantity.

In this paper, we attempt to incorporate the in-store handling cost, when not all the units of a product that a retail store orders and receives from a delivery can fill the shelf, into a replenishment decision model. This paper is the first to introduce the consideration of such in-store handling cost by the second trip. We will develop an EOQ-type model to determine the optimal order quantity considering the second trip cost. In addition, the pack quantity (i.e. number of units in a pack) will also be considered as it generally constitutes the minimum order quantity and retailers generally order in multiple of packs.

2. Single-trip in-store replenishment

In this section, we discuss the in-store handling activities. Ideally, in terms of in-store handling, we want to have single-trip in-store replenishment. Namely, only a single trip is needed for a store worker to travel to fill the shelf. This can be achieved by having the lot size of the received product smaller than the shelf space less the number of units left on the shelf at the shelf replenishment time. On the other hand, in case not all the units of a product that the store receives from a delivery can fill the shelf, the second trip is needed for the worker to travel between the store's backroom to the shelf. Fig. 1 illustrates the in-store handling activities considering number of trips traveled by a worker.

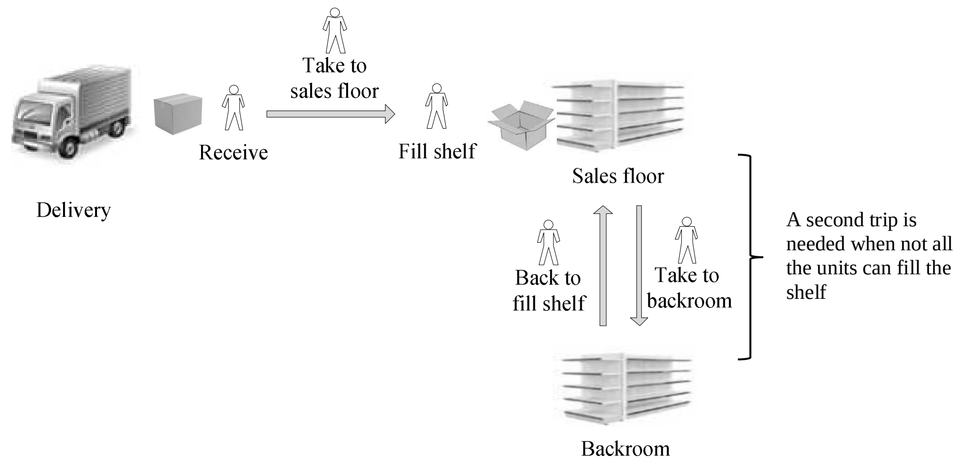

Fig. 1. The in-store handling when the second trip is needed.

3. Model development

In this section, we develop an EOQ-type model to determine the optimal order quantity considering the second trip cost. In our model:

(1) In case not all the units of a product (lot size) can fill the shelf, the second trip is needed and the cost of the second trip is constant for all shelf locations.

(2) We consider the case that a product is ordered in multiple of packs and the pack quantity is a decision variable.

(3) We consider the part of the distribution-handling cost that is per each pack handled and assume it is constant.

(4) The shelf space, which is in multiple of product units, is given for each product.

(5) The usual EOQ assumptions: infinite horizon, continuous and constant demand, instantaneous replenishment to store, and no shortage allowed, apply (see e.g., Nahmias, 1997).

Let us define the following notation.

q	Order quantity (in multiple of packs)
h	Holding cost per unit per year
k	Order set-up cost
s	Shelf space (units)
m	Pack quantity (units per pack)
c_d	Distribution handling cost per pack
D	Demand rate (units per year)
c_s	In-store handling cost for the second trip

Let us define the annual replenishment cost to consist of the order set-up cost, holding cost, distribution cost, and the store handling cost, and it is given by

$$G(q,m) = k\frac{D}{qm} + h\frac{qm}{2} + c_d\frac{D}{m} + c_s\beta(qm,s)\frac{D}{qm} \tag{1}$$

where

$$\beta(qm,s) = \begin{cases} 1; & \text{if } qm > s \\ 0; & \text{otherwise} \end{cases} \tag{2}$$

Then, let us define *Problem EOQSH1* (Economic Order Quantity with Second Trip Cost) as follow.

EOQSH1:

min $G(q,m)$,

subject to $m \geq 0$ and $q = 1, 2, 3, \ldots$

Let Q be the order quantity in units or $Q = qm$; $q = 1, 2, 3, \ldots$ and rewrite equation (1) as

$$G(Q,m) = k\frac{D}{Q} + h\frac{Q}{2} + c_d\frac{D}{m} + c_s\beta(Q,s)\frac{D}{Q} \tag{3}$$

Property 1

The optimal solution for *Problem EOQSH1* is such that $Q^* = m^*$ or $q^* = 1$.

Proof

Suppose the optimal $Q^* = q^* m^*$ where $q^* > 1$

Let m' be such that $Q^* = q^* m^* = (q^* - 1)m'$. This implies that $m' > m^*$.

Then, from equation (3), the total annual cost is given by

$$G(Q^*, m') = k\frac{D}{Q^*} + h\frac{Q^*}{2} + c_d\frac{D}{m'} + c_s\beta(Q^*,s)\frac{D}{Q^*} \tag{4}$$

Since $m' > m$, we have

$$G(Q^*, m') \leq G(Q^*, m^*) \tag{5}$$

a contradiction.

From Property 1, we can reduce *Problem EOQSH1* to

*EOQSH*2:

min $G(Q)$,

subject to $Q \geq 0$,

where

$$\begin{aligned} G(Q) &= k\frac{D}{Q} + h\frac{Q}{2} + c_d\frac{D}{Q} + c_s\beta(Q,s)\frac{D}{Q} \\ &= \left(k + c_d + c_s\beta(Q,s)\right)\frac{D}{Q} + h\frac{Q}{2} \end{aligned} \tag{6}$$

Because of $\beta(Q, s)$, $G(Q)$ is not a continuous function. To solve for the optimal order quantity, Q^*, we write equation (6) as the following two piecewise continuous functions. Fig. 2 illustrates the two piecewise functions of $G(Q)$.

Over $0 \leq Q \leq s$, $G(Q)$ reduces to

$$G_1(Q) = (k + c_d)\frac{D}{Q} + h\frac{Q}{2} \tag{7}$$

and Over $Q > s$, $G(Q)$ reduces to

$$G_2(Q) = (k + c_d + c_s)\frac{D}{Q} + h\frac{Q}{2} \tag{8}$$

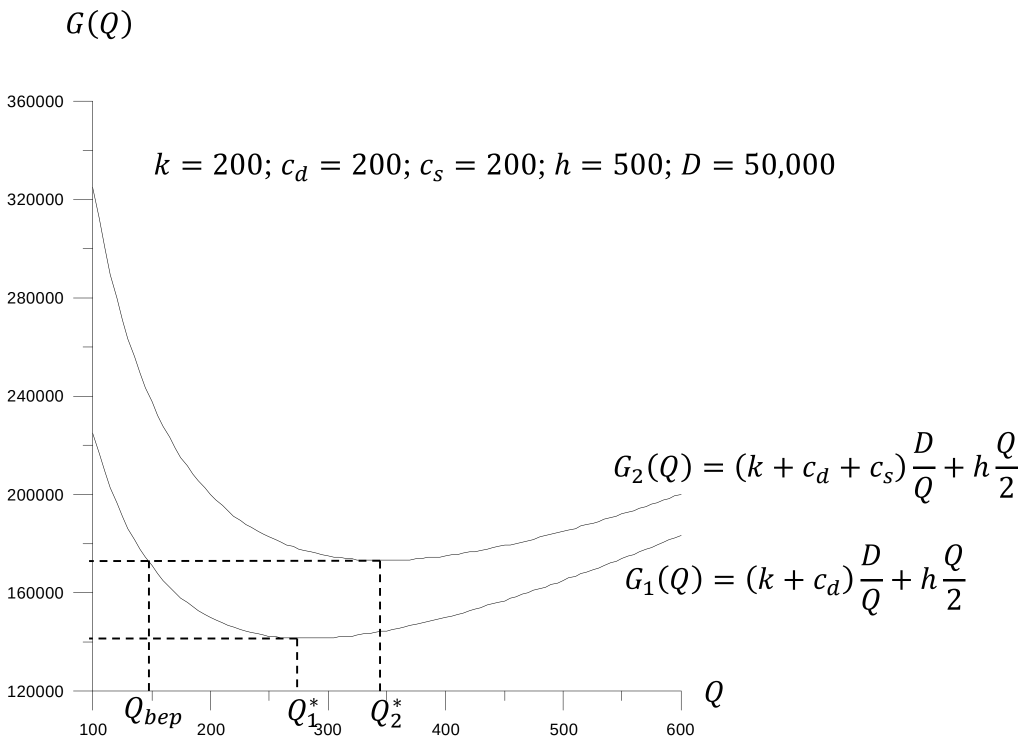

Fig. 2. Annual replenishment cost function

Let Q_1^* and Q_2^* be the stationary points of $G_1(Q)$ and $G_2(Q)$, respectively. We state the following properties. Some proofs are not provided since they are straight forward from the above discussion.

Property 2

The stationary point of $G_1(Q)$ is given by

$$Q_1^* = \sqrt{\frac{2(k + c_d)D}{h}} \tag{9}$$

and we say that $Q_1^*$is realizable if $0 \leq Q_1^* \leq s$.

Property 3

If Q_1^* is realizable, then the minimizer of $G(Q)$ is Q_1^*.

Property 4

If Q_1^* is not realizable, then $Q_1 = s$ is the minimizer of $G_1(Q)$.

Property 5

The stationary point of $G_2(Q)$ is given by

$$Q_2^* = \sqrt{\frac{2(k + c_d + c_s)D}{h}} \tag{10}$$

and we say that Q_2^* is realizable if $Q_2^* > s$.

Property 6

If Q_1^* is not realizable, then Q_2^* is realizable, and either Q_2^* or s, or both minimize $G(Q)$.

Proof

First, for Q_2^* to be realizable, we need to show that $Q_2^* > s$. Since $c_s \geq 0$, it follows from equations (7), (8), (9), and (10) that $G_1(Q) \leq G_2(Q)$ and $Q_1^* \leq Q_2^*$. Because Q_1^* is not realizable, it implies that $Q_1^* > s$. It then follows that $Q_2^* > s$. Then, since Q_2^* is realizable and is the minimizer of $G(Q)$, $Q > s$. In addition, from Property 4, $Q_1 = s$ is the minimizer of $G(Q), 0 \leq Q \leq s$, and this completes the proof.

Combining the results from Properties 2 to 6, the following corollary provides the solution for *Problem EOQSH*2.

Corollary 1.

The following algorithm optimally solves *Problem EOQSH*2.

1. Compute Q_1^*. If $Q_1^* \leq s$, then the optimal solution, $Q^* = Q_1^*$; otherwise, go to step 2.
2. Compute Q_2^*. If $G_2(Q_2^*) \leq G_1(s)$, then the optimal solution, $Q^* = Q_2^*$; otherwise $Q^* = s$.

Then, from Property 1, we have $m^* = Q^*$.

4. Implications in retail replenishment decisions

By considering the second-trip cost for the in-store replenishment and the distribution handling cost per pack, our model leads to the following implications.

(1) The in-store handling cost for the second trip, c_s, and the shelf space, s, impact the decision about the optimal order quantity, Q^*. Hence, retailers should consider the relationship between c_s, s, and Q^* when making the retail replenishment decisions.

(2) The lowest annual replenishment cost occurs when Q_1^* is realizable. That is, the shelf space, s, is greater than or equal Q_1^*. Therefore, when making the decision about the shelf space allocation, retailers should consider giving enough space to accommodate Q_1^*. In addition, when a retailer allocates shelf space to a product above Q_{bep}, the larger space will not impact the decision about the optimal order quantity.

(3) When the shelf space, s, is smaller than Q_1^*, the optimal annual replenishment cost increases while s decreases. However, while s gets below the breakeven point, Q_{bep}, as illustrated in Fig. 2. and determined by equation (11), the optimal annual cost becomes constant.

$$G_1(Q_{bep}) = \sqrt{2(k + c_d + c_s)Dh} \tag{11}$$

This means that when a retailer allocates shelf space to a product below Q_{bep}, the smaller space will not impact the decision about the optimal order quantity.

(4) When there is a part of the distribution handling cost per pack, for example the cost per each pack lifted or moved by a worker, which is generally the case in retail replenishment processes, the optimal pack quantity m^* is the same as the optimal order quantity, Q^* (Property 1). Therefore, retailers and manufacturers should work together to agree on the pack quantity by considering Q^*.

5. Conclusion and future research

This paper extends the EOQ model to include the consideration of the in-store handling cost by the second trip in case not all the units can fill in the shelf given a shelf space and the distribution handling cost per units. We developed an algorithm to optimally solve for the optimal order quantity. In addition, we discussed the implications in retail replenishment decisions for retailers and manufacturers from the results of our model development.

The contribution of this paper is significant because it is the first to introduce the concept of single-trip in-store replenishment and incorporate the consideration of the in-store handling cost by the second trip in a replenishment decision model. Many interesting research topics can be extended, and they include:

(1) To develop the rounding rule for rounding the order quantity to an integer number.
(2) To develop a replenishment decision model for the case that the pack quantity is given and the order quantity is in multiples of packs.
(3) To incorporate the inventory control decisions under periodic review such as the order interval and order-up-to point.
(4) To generalize the model to use the cost per trip multiplied by number of trips required as a function of order quantity instead of the second-trip cost.
(5) To develop a replenishment decision model for the case of uncertain demand.
(6) To incorporate the concept of single-trip in-store replenishment into a joint optimization model for inventory replenishment and shelf space allocation.

References

Corbett, C. J. (2000), “A supplier’s optimal quantity discount policy under asymmetric information”, *Management Science*, Vol. 46, No. 3, pp. 444-450.

Corstjens, M. and Doyle, P. (1981), “A model for optimizing retail space allocations”, *Management Science*, Vol. 27, pp. 822-833.

Datta, T. K. and Pal, A. K. (1990), "A note on an inventorymodel with inventory level demand rate", *Journal of the Operational Research Society,* Vol. 41, pp. 971-975.

Gerchak, Y., and Wang, Y. (1994), "Periodic-review inventorymodels with inventory-level-demand rate", *Naval Research Logistics,* Vol. 41, pp. 99-116.

Giri, B. C., Pal, S., Goswami, A. and Chaudhuri, K. S. (1996), "An inventorymodel for deteriorating items with stock-dependent demand rate", *European Journal of Operational Research,* Vol. 95, pp. 604-610.

Goyal, S. K. (1976), "An integrated inventory model for a single supplier-single customer problem", *International Journal of Production Research,* Vol. 15, pp. 107-111.

Hariga, M. A., Al-Ahmari, A. and Mohamed, A.-R. A. (2007), "A joint optimisation model for inventory replenishment, product assortment, shelf space and display area allocation decision", *European Journal of Operational Research,* Vol. 181, pp. 239-251.

Khmelnitsky, E., and Gerchak, Y. (2001), "Optimal control approach to production systems with inventory-level-dependent demand", *IEEE Transactions on Automatic Control,* Vol. 47, No. 3, pp. 289-292.

Lee, C. Y. (1986), "The economic order quantity for freight discount costs", *IIE Transactions,* Vol. 18, No. 3, pp. 318-320.

Lee, H. L., and Rosenblatt, M. J. (1986), "A generalized quantity discount pricing model to increase supplier's profit", *Management Science,* Vol. 32, pp. 117-1185.

Mandel, B. N. and Phaunjdar, S. (1989), "An inventorymodel for deteriorating items and stock dependent consumption rate", *Journal of the Operational Research Society,* Vol. 40, pp. 483-488.

Monahan, J. P. (1984), “A quantity discount pricing model to increase vendor profits”, *Management Science,* Vol. 30, pp. 720-726.

Nahmias, S. (1997), *Production and Operations Analysis* (3rd ed.), Irwin/McGraw-Hill.

Pal, S., Goswami, A. and Chaudhuri, K. S. (1993), “A deterministic inventorymodel for deteriorating items with stock-dependent demand rate”, *International Journal of Production Economics*, Vol. 32, pp. 291-299.

Saghir, M., and Jonson, G. (2001), “Packaging handling evaluation methods in the grocery retail industry”, *Packaging technology and science,* Vol. 14, No. 1, pp. 21-29.

Tersine, R. J. and Barman, S. (1994), “Optimal lot sizes for unit and shipping discount situations”, *IIE Transactions*, Vol. 26, pp. 97-101.

Toptal, A. (2009), “Replenishment decisions under an all-units discount schedule and stepwise freight costs”, *European Journal of Operational Research,* Vol. 198, pp. 504-510.

Toptal, A., Çetinkaya, S. and Lee, C.-Y. (2003), “The buyer–vendor coordination problem: Modeling inbound and outbound cargo capacity and costs”, *IIE Transactions on Logistics and Scheduling*, Vol. 35, pp. 987-1002.

Urban, T. L. (1992), “An inventorymodel with an inventory-level-dependent demand rate and relaxed terminal conditions”, *Journal of the Operational Research Society*, Vol. 43, pp. 721-724.

Urban, T. L. (1995), “Inventory models with the demand rate dependent on stock and shortage levels”, *International Journal of Production Economics,* Vol. 40, pp. 21-28.

Urban, T. L. (1998), “An inventory theoretic approach to product assortment and shelf space allocation”, *Journal of Retailing*, Vol. 74, pp. 15-35.

Wang, Y. and Gerchak, Y. (2001), “Supply chain coordination when demand is shelf-space dependent”, *Manufacturing and Service Operations Management,* Vol. 3, No. 1, pp. 82-87.

Zelst, S. v., Donselaar, K. v., Woensel, T. v., Broekmeulen, R. and Fransoo, J. (2009), “Logistics drivers for shelf stacking in grocery retial stores: potential for efficiency improvement”, *International Journal of Production Economics*, Vol. 212, pp. 620-632.